

# On the Optimal Choice of Operating Points in Large Series Hybrids

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## Abstract

This paper investigates some fundamental relationships for the optimal choice of operating points of ICEs in large series hybrids. It approaches the fuel minimization problem under the assumption that the engine cannot be switched off. The analysis focuses on the two operating point case with linear and quadratic break specific fuel consumption (bsfc) dependencies. Generalizations of these results to nonlinear (and non-quadratic) dependencies will be performed using a bounding argument. Explicit fuel saving conditions are given for the linear case and the amount of fuel savings will be quantified.

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## 1 Introduction

It is well known that large scale Diesel engines are prime candidates for hybridization using the series hybrid principle [1]. Especially for applications where power draw is almost periodic (like in heavy earth moving equipment or mining tasks) the series hybrid principle is a very attractive option. [2]

If energy storage capacity would be free or available at low cost, one could operate the ICE at an operating point that corresponds to the average required power over one cycle, while letting the storage device handle the short term mismatch of power. The disadvantage of such a scheme is not only the required large storage capacity, but also the fact that this average power is likely to not correspond to the lowest achievable *bsfc*, even though one can choose the operating point with the lowest achievable *bsfc* for this particular power level. Therefore an interesting alternative is the use of a two operating point (OP) scheme, with one OP being

the *bsfc* minimum. Assuming that the average power of a cycle is lower than the power that corresponds to the *bsfc* optimal power (which is the case in most applications), the other OP (that does not correspond to the minimum *bsfc*) must be a low power OP.

Slipstream Projects has introduced an optimal method in [3] by choosing this second operating point to correspond to "engine-off" conditions, effectively pulsing the engine between optimal *bsfc* and switching it off. This method was shown to produce superior fuel consumption results in a variety of drive cycles [4]. In large displacement Diesels, especially if the off state lasts on the order of minutes or even seconds, this is not possible [2]. We therefore investigate under which conditions it is advantageous to use a two operating point scheme relative to the one operating point (average power) scheme by using approximations of the *bsfc* versus power curve. It will be shown that if the *bsfc* optimal OP and the second low power OP are separated by some minimum distance then the two OP scheme is always preferable. We will

also show that in the case of the average power point being far below the  $bsfc$  optimal power, a two OP operating scheme is always preferable even if the high power OP is not  $bsfc$  optimal using a linear approximation. In the development of the results of this paper, we assume stay times in the two operating point to be sufficiently long to not affect the results due to transient phenomena that cause increased fuel consumption, and we assume the series hybrid drivetrain to have constant efficiency regardless of the power levels produced. (The latter condition can easily be lifted by including the driveline efficiencies in the analysis.)

## 2 Analytical Preliminaries

In this paper we will make heavy use of the relationship between optimal break specific fuel consumption ( $bsfc$ ) and power. This function, which we call,  $bsfc(P)$  is generated from the speed-power or the speed –torque diagram. For example, in the speed –torque diagram, given the power  $P$ , there are infinitely many points  $(T, \omega)$  that satisfy  $P = T\omega$ , where  $\omega$  is engine speed in radians per second and  $T$  is torque. Each of these points generates a  $bsfc$  value. The function  $bsfc(P)$  therefore maps power to the optimal (minimal) achievable  $bsfc$  at that power  $P$ , i.e. assigns to a power level  $P$  the minimal achievable  $bsfc$  value. This function  $bsfc(P)$  typically has a minimum in the mid speed range and increases for low and high engine speeds. Except for very low and very high power levels, this function is often well approximated by a positive quadratic function of the type:

$$bsfc(P) = k(P - P_{opt})^2 + bsfc_{min} \quad (1)$$

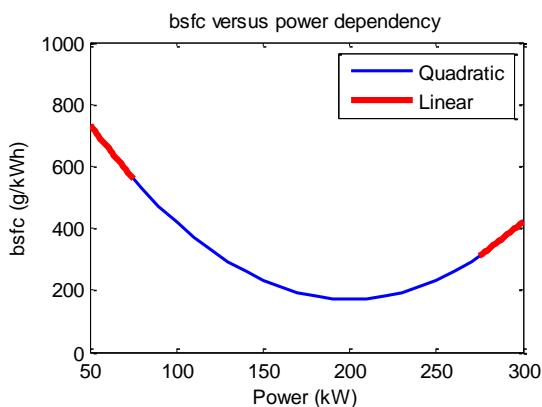


Figure 1: Quadratic and linear approximations to  $bsfc$  versus power dependency

$P_{opt}$  is the power for which the minimum  $bsfc$  is reached ( $bsfc_{min}$ ) and  $k$  is a positive constant. For operating ranges that correspond to a very high and a very low power level, we introduce a linear approximation of the form

$$bsfc(P) = bsfc_{max} + c P \quad (2)$$

Where  $c$  is a negative constant for small  $P$  and is positive for large  $P$ . In both cases, this approximation is only valid for a relatively small range of very low or high power levels. However, bounding techniques can be used in conjunction with this representation to obtain insights into the utility of two operating point cycling even if neither a quadratic nor a linear dependency exists. We also assume in this paper, that there exists a short and a long term average power, which are approximately equal. This power level is called  $P_{avg}$  and typically characterizes mining and earth moving type of duty cycles. In principle the analysis is valid also for cases where long term and short term averages are different, but it requires a more complex analysis taking different short term averages into account.

## 3 Fuel Saving Conditions

In this section we will provide conditions for an efficiency increase by cycling the engine using two operating points and the quadratic  $bsfc$  dependency in (1).

Consider the fuel mass equation:

$$M = \int_0^{P_{max}} p(P) bsfc(P) T_{total} dP \quad (3)$$

where  $p(P)$  is the probability of the engine producing power at level  $P$  and  $T_{total}$  is the total run time.

This equation does not consider the additional fuel consumption due to transients, i.e. it assumes a rather long time  $T_{total}$  and few changes in operating points. Of course, the fewer operating point changes and the larger  $T_{total}$  the better the approximation for the fuel mass using equation (3). In the following derivations we will use two different  $bsfc$  dependencies. At first we will investigate a quadratic relationship of  $bsfc$  versus power, in the second case a linear relationship, and then we will make a simple bounding argument using the result for the linear case to shed light on cases that are not linear. We would like to mention at this point, that when we refer to a linear or a quadratic relationship, we really only require that the three operating points satisfy the linear or quadratic relationship. These three points are the low power OP (with power  $P_1$ ), the high power OP (or the optimal  $bsfc$  OP in the quadratic case with

power  $P_2$  or  $P_{opt}$ ), and the average power OP (with power  $P_{avg}$ ). Therefore  $P_1 < P_{avg} < P_{opt}$ . Therefore this analysis is applicable to a large set of  $bsfc$  curves and linear and quadratic relationships are essential only for the three operating points, not the entire function or even an interval of the  $bsfc$  function.

### 3.1 The Quadratic Case:

In order to show one of our main results, we are setting up a fuel mass consumption equation of the following type:

$$M_{total} = T[qP_1bsfc(P_1) + qP_{opt}bsfc(P_{opt}) + (1 - 2q)P_{avg}bsfc(P_{avg})] \quad (4)$$

With  $P_{opt} - P_{avg} = P_{avg} - P_1 = \varepsilon$  and  $0 < q < 0.5$ , in this equation regardless of the parameter  $q$ , the average power is constant and equals  $P_{avg}$ . However if  $q$  is near 0.5, power is generated predominantly by cycling between the  $bsfc$  optimal OP and the low power OP, whereas if  $q$  is low, power is predominantly generated by a single operating point with power  $P_{avg}$ . Taking the derivative of (4) w.r.t.  $q$ , one obtains:

$$\frac{dM_{total}}{dq} = k(P_{avg} - P_1)^2(2P_1 - P_{avg}) \quad (5)$$

This derivative needs to be negative if cycling between the two OPs is to result in less fuel consumption. Since  $k$  and  $\varepsilon$  are positive,  $P_1 < \varepsilon$ , results in a negative derivative. It is now obvious, that for a quadratic  $bsfc$  relationship one can reduce fuel consumption (relative to the constant output power case of  $P_{avg}$ ) by cycling the engine between operating points at power level  $P_1$  and  $P_{opt}$  if the following condition is satisfied:

$$P_{opt} - P_{avg} > P_1 \quad (6)$$

This condition therefore requires that there is some minimum distance between  $P_{opt}$  and  $P_{avg}$  which is given by the low power operating point with power level  $P_1$ . Therefore it is advantageous if this low power operating point is chosen close to idle conditions. One can also see that if  $P_{avg}$  is close to  $P_{opt}$  then it may be hard to satisfy this condition. The amount of savings, which depend on the three power levels  $P_{avg}$ ,  $P_{opt}$  and  $P_1$  is given by

$$\Delta M = TP_{avg}bsfc(P_{avg}) - 0.5T[P_1bsfc(P_1) + P_{opt}bsfc(P_{opt})] \quad (7)$$

### 3.2 The Linear Case:

In the case of a linear dependency of the  $bsfc$  curve with a negative coefficient  $c$  in (2), it is easily shown that for  $P_1 < P_{avg} < P_2$ , where  $P_2$  does not necessarily have to correspond to  $P_{opt}$ , cycling between power level  $P_1$  and  $P_2$  is always advantageous over a single operating point with power level  $P_{avg}$ . This can be shown in a similar fashion as in the quadratic case, i.e. with equations (2) and (4). The resulting condition for the derivative in (4) is given by:

$$P_{avg}\varepsilon(P_1 - P_{opt}) < 0$$

Which with the definitions of  $\varepsilon$ ,  $P_{avg}$ ,  $P_1$ , and  $P_{opt}$  is always satisfied. Equation (7) also describes the fuel savings for the linear case. (We should point out that the expression “linear case” is somewhat of a misnomer, because the only requirement is that the three operating points lie on a straight line with negative slope, and this line never includes the origin.) In this particular case, the expression for (7) on the amount of saved fuel mass can be simplified to:

$$\Delta M = (P_{avg} - P_1)(bsfc(P_1) - bsfc(P_{avg}))T \quad (8)$$

Therefore the larger the difference between the average power and the two operating points in the cycle as well as the difference in the  $bsfc$  values between the average power point and the  $bsfc$  of the two operating points, the larger the fuel savings. Savings are proportional to the slope of the  $bsfc$  line, if all power levels stay the same.

### 3.3 Other Dependencies:

In the above two cases (linear and quadratic) we used the fact that the  $bsfc$  operating points for  $P_1$ ,  $P_{avg}$ ,  $P_2$  or  $P_{opt}$  satisfy a linear or quadratic relationship. Therefore it is not necessary that the entire  $bsfc$  curve satisfies a linear or quadratic dependency. The results for the quadratic and linear case above can be applied to any  $bsfc$  curve if there exist three points on this curve that satisfy the necessary conditions given above. The question that arises therefore is: what can one say about the advantages of cycling if the dependency is neither quadratic nor linear? The two cases for which this question can easily be answered using the developed results are a sublinear relationship (a relationship where the power  $P_1$  is below the line defined by  $(P_{avg}, bsfc(P_{avg}))$  and  $(P_2, bsfc(P_2))$ ) and subquadratic  $bsfc$  growth.

For the linear  $bsfc$  dependency it is easily shown that cycling is always advantageous, regardless of the relationship between  $P_1$ ,  $P_{avg}$  and  $P_{opt}$ . In a

“sub-linear” relationship, the value of  $bsfc(P_1)$  is actually lower than the one in a linear relationship, and hence the fuel consumption at power level  $P_1$  will be less. Since the stay time at the two operation points  $P_1$  and  $P_{opt}$  will be identical to the linear case in order to achieve a power average of  $P_{avg}$ , the overall fuel consumption will be less than in the case of linear dependency. Therefore in the case of a sub-linear  $bsfc$  dependency, cycling is advantageous, but can result in a small amount of fuel savings if the slope (given by  $c$ ) is small. In the case of a  $bsfc$  dependency that has a growth rate (in direction of diminishing power) that is between linear and quadratic, the condition  $P_{opt} - P_{avg} > P_1$  still guarantees that cycling results in less fuel consumption, for the same arguments as in the sub-linear growth case.

## 4 Conclusions and Outlook

This paper compares the fuel economy of operating a large Diesel in two different operating regimes: One operating regime is simply running the engine at a constant operating point at the average required power level; the other consists of cycling the engine between two operating points, one being the  $bsfc$  optimum. In both cases the generated average power is identical. Such an analysis is important for large Diesels that generate power in a series hybrid configuration and cannot be switched on and off over a short time period.

The analysis in this paper shows under which conditions it is advantageous to cycle an ICE. We distinguished between four cases: (1) quadratic  $bsfc$  dependency, (2) sub-quadratic  $bsfc$  dependency, (3) linear  $bsfc$  dependency, and (4) sub-linear  $bsfc$  dependency. In the first case, i.e. case (1), the three OP points lie on a quadratic and fuel savings are guaranteed only if the distance between the low and the average power point as well as the  $bsfc$  optimal and the average point are sufficiently large. Therefore it is important that the low power operating point generates as little power as possible and that the  $bsfc$  optimal power is significantly higher than the average power. In other words, if the average power and the  $bsfc$  optimal power are close, cycling should be avoided. In case of sub-quadratic  $bsfc$  dependency (case (2)) the condition for the quadratic case also ensures fuel savings for case (2). In case (3), cycling is always an advantage regardless of the

relationship between the three power levels, (always assuming that the  $bsfc$  optimal power is larger than the other two). Finally in case (4), cycling is also an advantage in all cases, like in case (3).

Explicit formulas for the amount of saved fuel in the linear and quadratic case are also given. As shown in [5], these savings can be significant.

Of course, a number of questions arise, that remain unanswered in this paper. For example, the use of more than two operating points could be advantageous in certain situations, especially when the cost of energy storage is factored in. One could use multiple operating points that are placed strategically to match required and produced power levels in a way that creates only small mismatches, leading to small energy storage requirements and thus to a significant reduction in cost. The other question that arises from this work is the role of transients. This paper totally neglects the additional fuel cost due to transients by assuming they are small due to long stay times in the two operating points. This again requires a large energy storage capacity and causes a high storage cost. Therefore an important question is: where is the optimal balance between high transient fuel consumption and high storage element cost? These topics will be the focus of future research.

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