

## **A Speed Profile Optimization for Automated Electric Vehicles Based on Dynamic Programming**

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### **Summary**

Recently, as connected vehicle technologies such as Vehicle To Vehicle(V2V) and Vehicle To Infra(V2I) have been developed, vehicles can predict future driving information and the information can be used for the control. If the target distance and velocity are known based on connectivity technologies, the control that minimizes the energy required for driving can be implemented with optimal control theories. These advantages are effective in autonomous vehicles because the driver does not drive directly. For example, if the autonomous vehicle is able to recognize distance to traffic light and traffic signal information, the target distance and velocity of the vehicle can be determined and the energy for driving can be minimized based on optimal control theories such as dynamic programming and pontryagin's minimum principle. Nowadays, electric vehicles are attracting much attention in order to solve environmental problems. In such electric vehicles, the driving range is one of the most important factors for attracting customers and can be extended with optimal controls based on connectivity technologies. However, this problem has two states such as distance and velocity of the vehicle, and it needs very high computing power to solve this problem based on dynamic programming. So, in order to save the computing time, the optimization problem with two states should be defined as a dual problem with one state and constraints. In this paper, it will be introduced how to define the dual problem with one state and constraints and how to optimize the speed profile of automated electric vehicles based on dynamic programming

*Keywords: Dynamic Programming, Speed Profile Optimization, Electric Vehicles, Automated Vehicles, Optimal Control*

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### **1 Introduction**

As artificial intelligence and big data processing technologies have been advanced, Vehicle communication technologies also have been growing. The connected vehicle technologies such as Vehicle To Vehicle(V2V) and Vehicle To Infra(V2I), which is one of these technologies, have been studied in many research institutions and companies[1],[2]. The forward vehicle information like both the velocity and distance to front vehicle and infra information about traffic lights and speed cameras can be known based on these connectivity technologies. We can predict the future driving information such as target speed and distance of the vehicle

with the forward vehicle information and infra information. For example, if the traffic light information like signal changing time and distance from the vehicle, can be known, the target speed and distance also can be determined. In addition, if the future driving information is known, the energy for driving can be minimized based on optimal control theories such as dynamic programming and pontryagins minimum principle[3]-[6]. The optimal control theories generally need future driving information and, in our problem, there are two states such as the velocity and distance of the vehicle. But, in order to solve this problem based on dynamic programming, it needs high computing power. So, this optimization problem with two states should be defined as a dual problem with one state and constraints for saving computing time. Recently, many people get interested in electric vehicles to cope with environmental problems[7]-[9] and, in particular, the driving range is important to attract customers for electric vehicles. The optimal control minimizing the energy required for driving based on connectivity technologies can extend the driving range of electric vehicles and it would be useful for growing sales of electric vehicles. This paper is structured as follows: In Section 2, the powertrain system for electric vehicle and dynamic equations are presented. In Section 3, the optimal control concept is introduced to define the dual problem. Conclusions are presented in the last Section.

## 2 Vehicle Model

### 2.1 Battery Model

The battery model with an internal resistance is shown in Figure 1, where  $V_s$ ,  $R_i$ ,  $i_o$  and  $V_o$  are the open circuit voltage, internal resistance, output current and output voltage. A state equation of SOC can be expressed as follows[10]:

$$SOC = -\frac{V_s - \sqrt{V_s^2 - 4 \cdot R_i \cdot P_S}}{2 \cdot R_i \cdot Q_S} \quad (1)$$

where  $P_S$  and  $Q_S$  are the battery power and capacity of the battery.

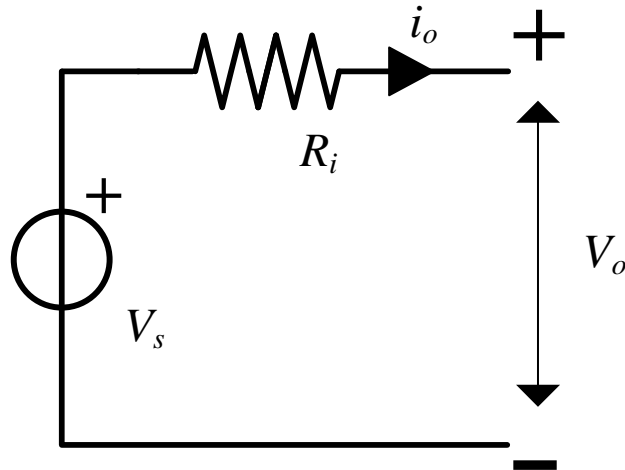


Figure1: Static battery model with an internal resistance

### 2.2 Powertrain System Configuration

The powertrain system of the electric vehicle has one gear and is shown in Figure 2. The motor's speed and torque can be calculated as follows:

$$S_{mot} = \gamma \cdot \zeta \cdot S_{out} \quad (2)$$

$$T_{mot} = \frac{1}{\gamma \cdot \zeta} \cdot T_{out} \quad (3)$$

where  $\gamma$ ,  $\zeta$ ,  $S_{out}$  and  $T_{out}$  are the gear ratio, final drive ratio, wheel speed and wheel torque.

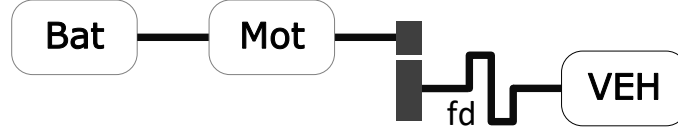


Figure2: Powertrain system of electric vehicles

### 3 Optimal Control Concept

#### 3.1 Dynamic Programming

Optimal control theories, such as dynamic programming and pontryagin's minimum principle, are to define cost function and minimize the cost function with state equations. Dynamic programming can derive a globally optimal solution whereas pontryagin's minimum principle can generally find a locally optimal solution with hamiltonian. However, dynamic programming requires high computing power and this phenomenon becomes worse as the dimension increases. When the target speed and distance are determined, the optimal control problem with two states can be defined as follows:

$$\min J = \int_{t_0}^{t_f} P_{bat}(v, u) dt \quad (4)$$

$$\text{subject to } \dot{v} = f(v, u) \quad (5)$$

$$\dot{z} = v \quad (6)$$

where  $v$ ,  $z$  and  $u$  are the vehicle's velocity, distance and motor torque. In order to decrease computing time, the dual problem with one state can be defined by considering the state of the distance as a constraint as follows:

$$\min L = \int_{t_0}^{t_f} \{P_{bat} + \lambda \cdot v\} dt \quad (7)$$

$$\text{subject to } \dot{v} = f(v, u) \quad (8)$$

where  $\lambda$  is called Lagrange multiplier or co-state.

#### 3.2 Simulation Results

In order to apply dynamic programming, the backward model is used for simulation. The simulation conditions and results are shown in Table1 and Figure 3. In Figure 3, the optimal speed profiles change according to the co-state and the driving distance also changes. The driving distance increases as the absolute value of the co-state increases.

Table1: Simulation Conditions with Dynamic Programming

	Simulation Time	Time Step	Initial Speed	Target Speed
Conditions	100s	0.1s	80km/h	80km/h

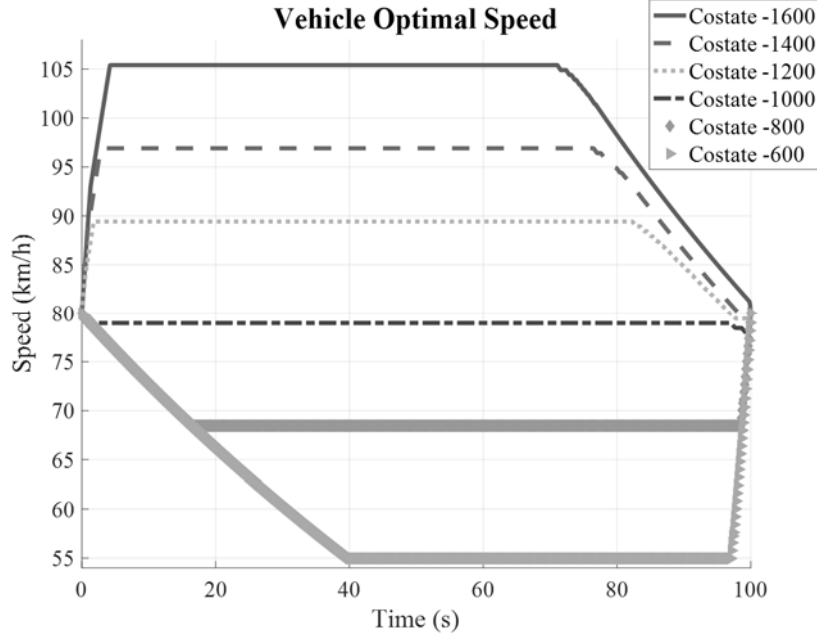


Figure3: Optimal speed profile according to co-states

## 4 Conclusion

Nowadays, connectivity technologies have been researched and it has become possible to recognize the information on both forward vehicles and infra. The future driving information such as the target speed and distance can be predicted with connectivity technologies and energy for driving can be minimized based on optimal control theories. This control strategy can be applied to autonomous vehicles because the vehicle can drive itself. This paper introduces the optimal control strategy that minimizes the energy of electric vehicles for driving based on dynamic programming. In order to decrease the computing time, the dual problem with one state is defined. The simulation result show that the optimal speed profiles and driving distance depends on the costate. In future work, it will be analyzed in detail why this speed profile is optimal with constraints such as driving distance and time.

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