

## Methods to evaluate the quality of a remaining range algorithm

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### Summary

Compared to diesel buses, the range of electric buses is severely limited. Therefore, accurate calculation of the remaining range is crucial. Today, the quality of the range calculation is often determined by comparing the total distance traveled with the absolute change of the calculation. The main problem with this approach is the lack of evaluation regarding the dynamics of interim calculations. Hence, we discuss and compare various criteria based on 25 test drives and 54 variations of an existing remaining range algorithm. A suitable method for evaluating range algorithms is to use MAPE or RMSE in combination with a quantile, we chose 0.1, to account for not only the best result with the lowest error but several good results for each test run.

*Keywords: bus, modelling, optimization, range, EV (electric vehicle)*

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### 1 Introduction

Public transport and, in particular, the use of electric city buses is expected to play a key role in the improvement of air quality in cities. One of the biggest challenges for the use of electric buses in public transport is the very limited range compared to diesel buses. Therefore, the correct remaining range prediction is essential. Accurate range prediction reduces range anxiety [1], [2]. If both the driver and operator trust the estimated remaining range, there is no need to keep a SOC buffer in the battery, which would further reduce the drivable distance. Reference [3] states that an accurate estimation of the remaining range therefore corresponds to an increase of the battery capacity by up to 30%.

Existing in-vehicle remaining range calculations are often based on easily accessible historical consumption values. However, a city bus operates for several hours a day during which the environmental conditions affecting consumption, such as outside temperature, number of passengers, traffic density or route topology, can change drastically. Therefore, the likelihood that historical values will always correspond to current consumption is low. But since the route of a city bus is already planned hours or even days before its start, a prediction of the driven route and its conditions can easily be carried out. Example approaches for such an energy consumption forecast to improve the remaining range calculation are the physical models in [3] or [4], the data based creation of a probability map with the marked reachable destinations based on route segments in [5] or the use of Markov chains or neuronal networks in [6] and [7].

Although there are many such approaches to forecasting the remaining range, methods for assessing the quality of these approaches are rarely discussed. Most references use one or more of these assessment

methods: “looking closely” at the measured vs. forecasted curve [8], absolute error (AE) [2] or relative error (RE) [3] of the start and end values, root mean square error (RMSE) [9], mean absolute percentage error (MAPE) [10], standard deviation [5] or R-squared [11]. However, it is not evaluated which of these methods is suitable for the evaluation of the remaining range calculation. While, on the one hand, the comparison of the curves is not handy if a higher amount of test drives is used for evaluation, on the other hand the absolute error of the end values does not allow any conclusion on the dynamic behavior of the range calculation during the drive. Thus, to be able to make a reliable statement about the benefits of newer and more complex approaches over existing ones, evaluation criteria that allow the comparison of the entire curve of the ride are essential. In addition, not only one test drive but several scenarios should be considered in the evaluation to ensure that the advantages or disadvantages found are not limited to specific environmental conditions.

This paper discusses various approaches to assess the quality of the remaining range algorithms based on 25 test drives of an electric city bus with a total distance of 3631 km. During these drives the environment and driving conditions vary significantly. Section 2 explains the challenges of the remaining range calculation of electric vehicles based on these test drives. Section 3 then discusses methods to evaluate such calculations. Section 4 shows the results obtained with the found evaluation method based on the described test data. The final section gives a conclusions and an outlook on future work.

## 2 Challenges of remaining range calculation and its evaluation

Reference [1] states that the aforementioned range anxiety in electric vehicles refers from the combination of short overall drivable distance combined with high recharging times and high range prediction errors. Further, the impact of calculation errors is higher than in a combustion engine vehicle, as the range of an electric drive is much shorter [3]. Therefore, a high accuracy is essential. But why is the remaining range calculation of an electric vehicle such a challenge, if it has been done for years for vehicles with internal combustion engine?

First of all, the achievable range depends on two factors: the usable on-board energy and the average consumption. While in a combustion vehicle only the consumption depends on environmental conditions and vehicle dynamics, in an electric vehicle also the usable energy is influenced by these parameters. Therefore, both the energy demand and the battery behavior must be accurately predicted, e.g. [12], [13] or [14], to calculate the remaining range of an electric vehicle. Reference [15] gives a good overview of the factors used for different range calculation approaches, such as route characteristics, weather, vehicle dynamics, traffic, given vehicle parameters and auxiliary consumers. In addition, with regard to auxiliaries, the influence of the heating system on the total consumption due to the lack of waste heat is considerably higher than for vehicles with internal combustion engines [16], [17]. Even more so in a city bus than in a car, as it opens its doors at every bus stop and therefore most of the thermal energy is lost through air exchange.

Secondly, in contrast to diesel buses, electric buses do not only consume energy but also generate energy through recuperation. Often evaluations compare the calculated remaining range with the measured driven distance, e.g. [3] and [8]. Basically this approach assumes that the range calculation is correct, if it behaves like the driven distance: for each driven kilometer the remaining range should be reduced by one kilometer. The problem with this assumption is the distance driven increases during recuperation and the remaining range does not decrease but rather increases due to the increase of stored energy on-board and the reduced energy consumption. Furthermore, this recuperation effects can significantly influence the dynamics of the range calculation, especially in hilly areas. A solution to this problem might be to compare not the driven distance but the change in present consumption to the dynamic of the calculated range. The remaining range gradient should decrease or become even negative with low consumption and increase with high consumption. But as the present consumption has a high dynamic, no one would rely on the range calculation if it changes accordingly. This would sabotage the goal of calculating a reliable remaining range. Therefore, the challenge is to give the driver a reliable and “stable” remaining range value while considering the dynamically changing consumption nonetheless.

This leads to our second question: what is the correct remaining range? Some approaches try to calculate the uncertainties in their performed range calculations due to dynamic influencing factors [12], [14]. In theory, this is great to get a sense of the influence of the different factors on the range calculation. In practice, the driver wants to know reliably how far he can still drive with his vehicle. His main concern is if he can reach

his destination without reloading and not how probable it is. So we are back to our problem how can we assess, which remaining range calculation is on the one hand the most precise and on the other hand the most reliable and should therefore be used in a bus to inform the driver.

## 2.1 Influence of test data on remaining range calculation

Since the energy consumption of an electric city bus is influenced by a wide range of static and dynamic parameters, a total of 25 test drives with a variety of routes, driver behaviour and environmental conditions are used as basis for finding a good evaluation method. This ensures that the advantages or disadvantages found are not limited to specific environmental conditions of the evaluation data. Table 1 shows specific characteristics of each test drive: the average speed  $v_{avg}$ , the average Temperature  $T_{avg}$ , if it has significant slope, if there is additional load in the bus, if additional auxiliaries like HVAC are used, the depth of discharge (DOD) of the usable battery energy during the trip and the driven distance  $s$ .

The average speed of the test drives is from 8 km/h to 65 km/h. The temperature ranges from winter scenarios with an average temperature of  $-3$  °C to summer scenarios with an average temperature of  $34$  °C. Further some test drives include altitude changes in the route topology, passenger load and the use of additional auxiliary units like the heating or door opening. We want to emphasize that there are eleven trips where we have completely emptied the battery in the end, ten with a DOD of 93% or higher. With these rides, we certainly know the drivable range. It varies with the average energy consumption during these trips between 150 km to 316 km. This shows the aforementioned significant impact of the given parameters on the drivable range of an electric vehicle.

Table1: Specific conditions of the different test drives used for the evaluation

No	$v_{avg}$ [km/h]	$T_{avg}$ [°C]	Slope [-]	Load [kg]	Auxiliary [-]	DOD [%]	$s$ [km]
1	37	24	-	-	yes	100	250
2	16	34	-	-	yes	20	52
3	26	31	-	-	-	43	154
4	42	30	-	-	-	96	295
5	49	32	-	-	-	96	281
6	42	27	-	-	-	94	152
7	43	30	-	-	-	100	312
8	28	28	-	-	-	100	316
9	42	20	yes	-	-	96	277
10	37	21	yes	-	-	100	283
11	65	19	-	-	-	100	283
12	23	29	-	-	yes	21	46
13	19	26	-	yes	yes	93	150
14	21	1	-	-	yes	60	74
15	27	-3	yes	-	yes	55	80
16	19	13	-	-	yes	45	79
17	19	16	-	-	yes	37	70
18	15	22	yes	-	yes	44	83
19	21	31	-	-	yes	64	102
20	17	26	-	-	yes	56	100
21	10	5	yes	yes	yes	52	44
22	9	7	yes	yes	yes	30	28
23	9	5	yes	yes	yes	45	38
24	9	6	yes	yes	yes	39	37
25	8	9	yes	yes	yes	43	45

Fig. 1 and Fig. 2 show plots of the inverse driven distance along with a corresponding remaining range calculation curve of different test drives of Table 2. The range calculation used is the same for all four plots shown. With the exception of test drive 25, the battery was completely discharged in all diagrams. Thus, for test drive 25 the drivable distance is not known. Therefore, we adjusted the end value of the inverse driven

distance to the end value of the range calculation in the diagram. Thus, it is not possible to compare start or end value reliably. However, the graph shows that the calculation reduces its values similar to the distance traveled. If we had just used test drive 13 and test drive 25 in Fig. 1 for validation, we would have assumed that the given range calculation is fairly reliable since both curves behave like the distance travelled. Furthermore, their difference in average speed, average temperature and weight does not seem to significantly affect the accuracy of the range calculation.

Comparing the diagrams in Fig. 2, this statement is refuted. First of all, test drive 10 shows the significant influence of hilly roads on the algorithm used. With the changing slope of the road, the calculated range changes within a 20km distance several times over 300km. In comparison test drive 11 seems more accurate but is still highly dynamic. In test drive 11 we see the influence of a dynamic speed profile with a high average speed and thus a high impact of the acceleration and deceleration on the range calculation. In addition, both graphs in Fig. 2 show high start errors due to low stored historic consumption values. No driver would rely on this range calculation.

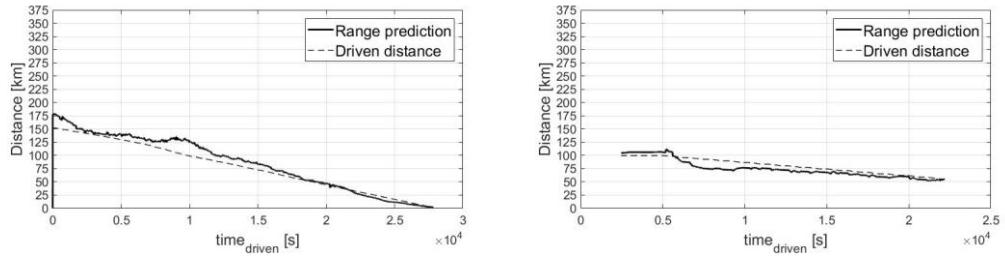


Figure1: Inverse distances and the corresponding remaining range calculation of test drive 13 (left) and 25 (right)

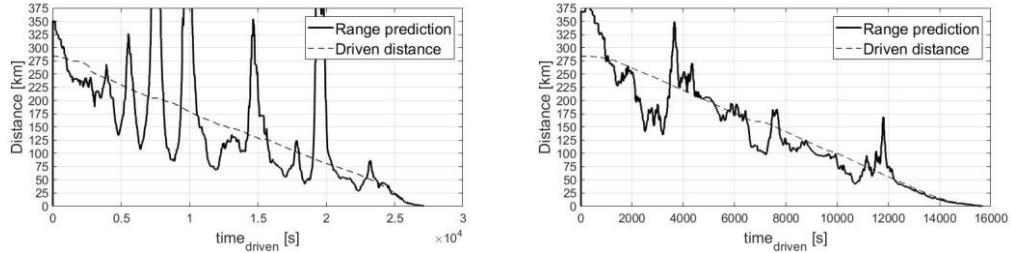


Figure2: Inverse distances and the corresponding remaining range calculation of test drive 10 (left) and 11 (right)

### 3 Methods for evaluation of remaining range algorithms

On the one hand, the question arises as to which methodology is best suited for comparing different range algorithms and, on the other hand, which quantities should be compared. The comparison between a measurement and a forecast curve like in [8] is not handy if a higher amount of test drives shall be used for evaluation. In order to be able to easily evaluate a large number of variations of one or more range algorithms, a single evaluation value is required for comparison. For example the absolute error (AE) between the total distance travelled and the gradient of the calculated start and end values in [2] not only indicates whether the driver could trust the given starting value, but also allows a simple comparison of a higher number of calculations. Its disadvantage is the lack of consideration of the dynamics of the calculation. If the difference of the considered start and end values coincides, this does not yet indicate that the range calculation is good overall.

Reference [18] discusses several measures of accuracy of univariate time series forecasts. Some are scale dependant like the (Root) Mean Square Error ((R)MSE) or the Mean Absolute Error (MAE). The RMSE is mostly preferred to the MSE as it is on the same scale as the data, but compared to the MAE this measure is more sensitive to outliers [18]. Another possibility are percentage errors, which are scale independent like the Mean Absolute Percentage Error (MAPE), the Median Absolute Percentage Error (MdAPE) or the Root Mean Square Percentage Error (RMSPE). The problem with these measures is that they are undefined in case of zero [18]. As the MAPE and MdAPE also put a heavier penalty on positive errors [19], exist for both symmetric variants which avoid this effect. However, this could even be an advantage for the range

calculation, since too small range values do not cause any breakdowns and are therefore better than too large ones. Finally there are relative error measures, which compare the error obtained with another forecasting method like the Mean Relative Absolute Error (MRAE) [18].

Table 2 shows an overview over the mentioned measures and their known use to asses remaining range forecasts. The formulas are defined as in [18] with  $Y_t$  being the measured value at time  $t$  and  $F_t$  being the forecast of  $Y_t$ . The error is  $e_t = Y_t - F_t$ . Percentage errors are calculated by  $p_t = 100e_t / Y_t$ . Table 2 shows that a wide range of these evaluation criteria has already been used to determine the accuracy of the range forecast. The focus, however, is on the absolute or relative error of the start and end values.

Table2: Different measures of accuracy of time series forecasts used for range forecast evaluation

Measurement	Abbr.	Formula	Used in...
Absolute Error (start and end values)	AE	$(Y_{\text{end}} - Y_1) - (F_{\text{end}} - F_1)$	[2], [3], [20]
Mean Absolute Error	MAE	$\text{mean}( e_t )$	[21]
Root Mean Square Error	RMSE	$\sqrt{\text{mean}( e_t^2 )}$	[21], [9]
Relative Error (start and end values)		$ p_t $	[3], [4], [5], [12], [22], [23], [24]
Mean Absolute Percentage Error	MAPE	$\text{mean}( p_t )$	[10]
Root Mean Square Percentage Error	RMSPE	$\sqrt{\text{mean}(p_t^2)}$	[25]
Symmetric Mean Absolute Percentage Error	sMAPE	$\text{mean}(200 e_t /(Y_t + F_t))$	[11], [26]
Mean Relative Absolute Error	MRAE	$\text{mean}( e_t/e_{t,basic} )$	-
Coefficient of determination (R-squared)	$R^2$		[7], [21], [11]

In addition to the best matching evaluation criteria, the question arises, which values should be evaluated. Often the curve of the range forecast is compared with the inverse driven distance. If the calculation behaves inversely to the driven distance, this is understandable for the driver and thus strengthens the feeling of reliability of the calculation. However, for the evaluation, this method is faced with the difficulty that it is not clear what the actual range would have been, if the battery had not been completely discharged. Moreover, we described in the previous chapter that due to the recuperated energy, the correct possible range does not have to strictly follow the route travelled. One possibility to improve would be a relative tolerance to ignore small differences or erase specific influences from the evaluation value. In order to overcome the problem of the unknown drivable distance in the case of energy left in the battery, the curve for the inverse driven distance could be placed in favour of a small deviation to the calculated curve. For each calculation, the best case would be assumed and compared with the others. Further, derivatives could be compared instead of the absolute values.

Furthermore, it may be useful to combine different evaluation criteria or forecast values. Instead of calculating only a total value for each evaluation, such as the MAPE, the interquartile range of the calculated individual error values  $e_t$  could also be taken into account in the assessment. The higher the interquartile range the lesser the coincidence of the curves. What is more, to overcome the recuperation challenge, comparing the difference of the distance to the inverse route could be combined with a comparison of the derivative of the distance with the current consumption or the SOC curve. However, this would make the evaluation approach quite complex. Therefore, in this article we focus solely on the comparison of the individual evaluation criteria from Table 2 except for the MRAE, which was never used for range evaluation, and the R-squared, as we did not use a statistical method for the calculation.

### 3.1 Comparison of the defined evaluation methods for the given test drives

For the comparison of the evaluation criteria, we evaluated a total of 54 range calculations for each test drive. The different calculations resume from changing four parameters of our existing range calculation algorithm. Three parameters, A, B and C, have three variations, while the last parameter D has two variations. As evaluation criteria we examine the AE, MAE, RMSE, RMSPE, MAPE and sMAPE of Table 2. Since it is customary to evaluate the range forecast with the inverse driven distance and we already discussed that such

a behavior is understandable for the driver, we stick to this approach, despite the difficulties discussed earlier. The 11 trips with completely discharged battery allow us to make a reliable statement about their possible driving distance. For the remaining trips, we set the final values of the distance traveled equal to the value of the range calculation to calculate the evaluation criteria. Finally, as a possible alternative method of evaluation, we also calculate a RMSE for the derivatives instead of the absolute values (RMSE<sub>diff</sub>). A selection of the curves belonging to the resulting minimum and maximum values of these evaluation criteria are shown in Fig. 3 to Fig. 5. In addition, the associated distance travelled is shown in each diagram.

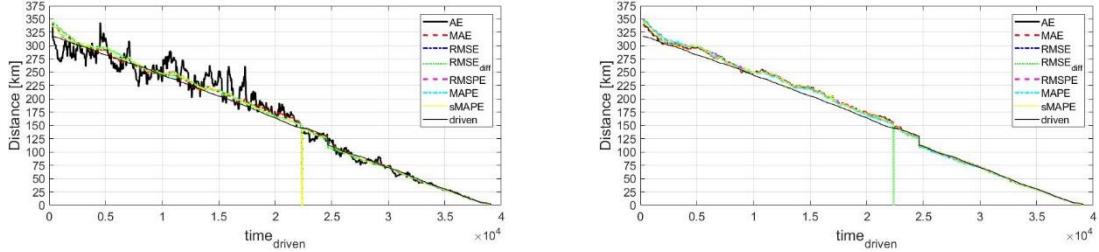


Figure 3: The result curves of the minimum (left) and maximum (right) of different evaluation criteria for test drive 8

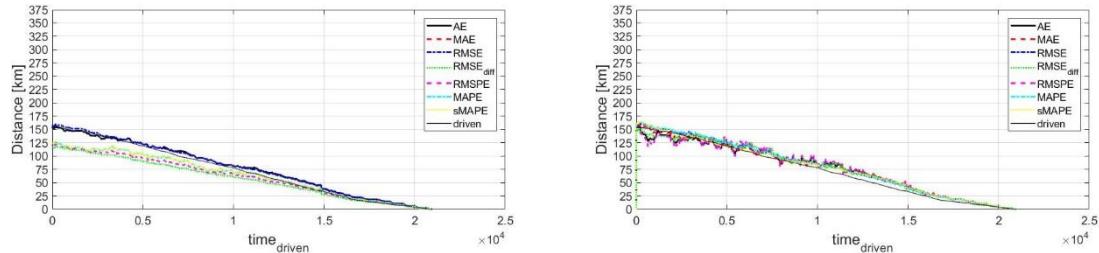


Figure 4: The result curves of the minimum (left) and maximum (right) of different evaluation criteria for test drive 3

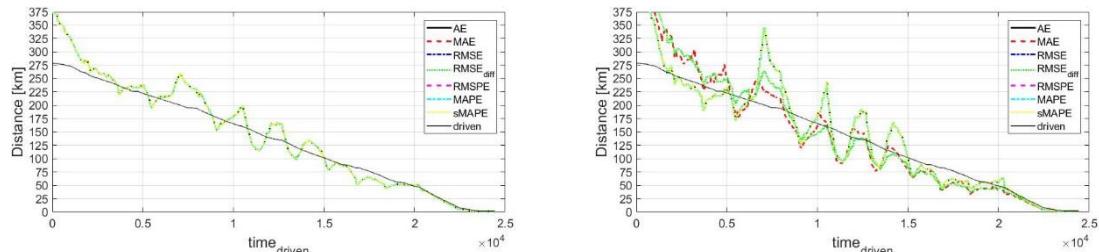


Figure 5: The result curves of the minimum (left) and maximum (right) of different evaluation criteria for test drive 9

Except for the test drives where all evaluation criteria result in the same range curve, see for example Fig. 5 left, the results of the AE usually differ from the results of the other evaluation criteria. In Fig. 3, for example, the results of all considered evaluation criteria are similar except for the minimum AE value (left). While for the minimum AE the difference of the start and end values of the remaining range curve obviously matches the inverse distance travelled, the dynamic behavior of both curves differ considerably. Conversely, the curves of the other evaluation criteria are usually the same as the distance travelled, but they show a higher error at the beginning of the calculation. This shows that despite a small AE, the dynamic behavior of both curves may differ considerably. Therefore, the comparison of the absolute value only or the corresponding relative error does not appear to be a good quality measure. Therefore, we do not consider the AE for the further evaluation.

The curves for the minimum and maximum MAPE and sMAPE are identical for nearly all considered test drives, see also Fig. 3 to 5. In the cases where they differ, the curves are either similar or the curve of the MAPE is closer to the distance travelled and the results of the other criteria. Further, the minimum RMSPE curves differ only in 5 cases from the MAPE and for these test drives the associated range curves are still very similar. Therefore, in the further discussion, only the MAPE will be used as a percentage error measure. The same applies to the curves of the minimum MAE in comparison to the RMSE. Hence, we choose the RMSE as second evaluation criterion for further discussion.

In addition, Fig. 5 particularly shows the difference between the best and the worst range algorithm considering the given evaluation criteria. The range curves of the minimum values still show some dynamics due to the recuperation, but they mainly follow the distance traveled. In contrast, the range curves of the maximum values have higher start errors, have a higher dynamic and are therefore unreliable. This leads to the conclusion that the discussed valuation methods are suitable for the valuation of the range calculation.

In order to assess whether these findings are also valid for the other 14 test drives with no discharged battery in the end, we perform the range calculation for the first ten test drives again with only the first half of the measured data. Fig. 6 shows the results for test drive 3 as example. We chose this method because, unlike the other shorter test drives, we can use the results for the correct driven distance as a comparison. As for all shorter test drives, we equate the two end values. The minimum values in Fig. 6 left lead to nearly the same overall driven distance than the original data in Fig. 4. However, the optimal curve found using the  $RMSE_{diff}$  differs significantly from the previous results as well as from the results of the other evaluation criteria. For the maximum value, the result deviates significantly from the other criteria, too. If we compare these results to the exemplary ones of test drive 12 in Fig. 7, the range curve of the minimum  $RMSE_{diff}$  again differs from the ones of all the other criteria. As in Fig. 6, the optimal algorithm found estimates less than half the range of the other results. In summary, we can say that although the evaluation of the gradient leads to curves with a similar dynamic behavior to the distance traveled, their absolute values can deviate significantly from the real values for shorter distances. Since the results of the  $RMSE_{diff}$  largely correspond to those of the other criteria in case of a high DOD and these criteria seem to provide robust results even for shorter distances, the  $RMSE_{diff}$  is not considered further.

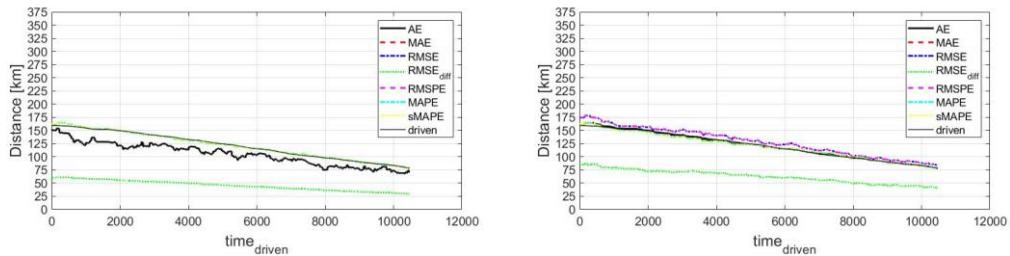


Figure 6: The result curves of the minimum (left) and maximum (right) of different evaluation criteria for only the first half of test drive 3

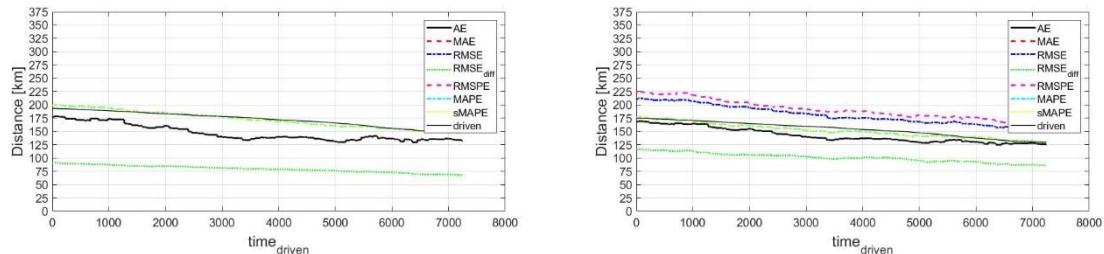


Figure 7: The result curves of the minimum (left) and maximum (right) of different evaluation criteria for test drive 12

### 3.1.1 Influence of test data on the evaluation result

Table 3 provides the parameter combinations which lead to a minimal calculated MAPE and RMSE value for the test drives of Table 1. First of all, both evaluation criteria differ considerably between the different test drives. The MAPE varies from 0.6% to 22.6%. The RMSE varies between 1.0 km and 23.4 km. This shows the significant effect of the different considered environment conditions of the test drives on the accuracy of the remaining range evaluation. Interesting is that the highest MAPE value occurs for test drive 7 while the highest RMSE value occurs for test drive 9. In addition, the optimal parameter combinations with minimal MAPE or minimum RMSE are not identical. Moreover, the results differ between these two criteria for several test drives. Therefore, we compared the MAPE and RMSE results for all 54 considered parameter combinations. Fig. 8 shows an example for the drives with the minimum and maximum values. We conclude that although the optimal combination of parameters found in Table 3 differs, the basic behaviour of the two evaluation criteria is the same across the 54 investigated combinations.

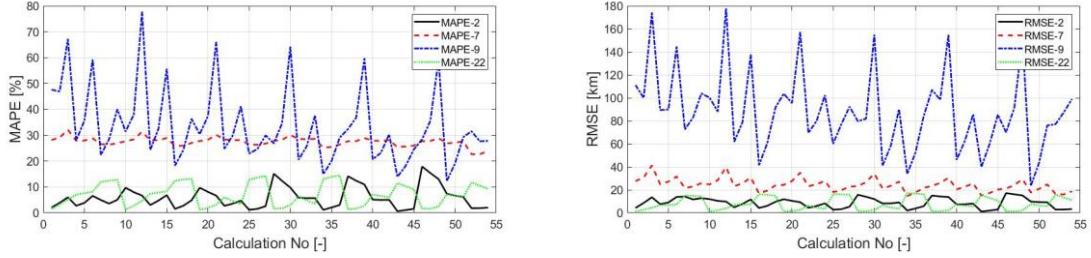


Figure8: MAPE (left) and RMSE (right) for the 54 different remaining range calculations for test drive 2, 7, 9 and 22

To account for these considerations, we consider not only the algorithm with the lowest MAPE or RMSE value of each test run, but the x-quantile of the lowest MAPE or RMSE results. Thus, not the algorithm with the lowest RMSE or MAPE value for a particular case is considered optimal, but the algorithm that provides the highest number of optimal values, taking into account the different environmental conditions of all the test drives used for the evaluation.

Table3: Parameter Combination chosen by the different evaluation criteria

No	MAPE min	Parameter min	RMSE min	Parameter min
1	9.0%	A1, B1, C3, D2	9.0km	A1, B1, C3, D2
2	0.6%	A3, B1, C3, D1	1.1km	A3, B1, C3, D1
3	14%	A1, B3, C3, D2	5.3km	A2, B1, C3, D2
4	8.7%	A2, B1, C2, D2	14.2km	A2, B1, C1, D2
5	3.8%	A2, B1, C2, D2	6.4km	A2, B1, C2, D2
6	9.4%	A1, B1, C3, D2	5.5km	A1, B1, C3, D2
7	22.6%	A3, B2, C3, D2	14.6km	A3, B1, C2, D2
8	2.9%	A1, B1, C3, D2	6.0km	A1, B1, C3, D2
9	12.3%	A2, B1, C3, D2	23.4km	A2, B1, C3, D2
10	10.6%	A3, B1, C2, D2	16.5km	A3, B1, C2, D2
11	9.7%	A3, B1, C2, D2	13.5km	A2, B1, C3, D2
12	1.5%	A3, B1, C3, D2	3.3km	A3, B1, C3, D2
13	9.4%	A1, B1, C3, D2	5.5km	A1, B1, C3, D2
14	2.0%	A2, B1, C2, D1	2.2km	A2, B1, C2, D1
15	2.1%	A3, B1, C2, D2	2.2km	A3, B1, C2, D2
16	2.3%	A3, B1, C2, D2	3.0km	A3, B1, C2, D2
17	1.8%	A3, B1, C3, D2	1.1km	A3, B1, C3, D2
18	4.8%	A1, B1, C2, D1	8.7km	A1, B1, C2, D1
19	2.5%	A2, B1, C1, D1	3.7km	A2, B1, C1, D1
20	3.1%	A3, B1, C1, D2	3.1km	A3, B1, C1, D2
21	4.9%	A1, B1, C1, D1	3.1km	A1, B1, C1, D1
22	1.2%	A1, B1, C2, D1	1.0km	A1, B1, C1, D2
23	1.7%	A1, B1, C2, D1	1.4km	A1, B1, C3, D1
24	2.2%	A2, B3, C3, D1	2.6km	A2, B3, C3, D1
25	3.4%	A2, B3, C3, D2	4.1km	A1, B1, C1, D1

#### 4 Results of the chosen evaluation method for remaining range algorithms

We divided the evaluation in two parts: first we just considered the eleven test drives with a known drivable range and then we used all given test data for the evaluation. For the eleven test drives with a discharged battery we chose the 0.1-quantile to consider not only the best parameter combination of each test drive in the evaluation but the lowest 5 values. The scatter plots in Fig. 9 show that for both the MAPE and the RMSE the results of test drive 9 are high and the results of test drive 5 and test drive 8 are low. It is interesting that Test Drive 7 also has quite high MAPE values, while it has rather low RMSE values. Fig. 8 shows that the results of the evaluation criteria for the algorithm variations are basically the same, only the total value of the error varies significantly.

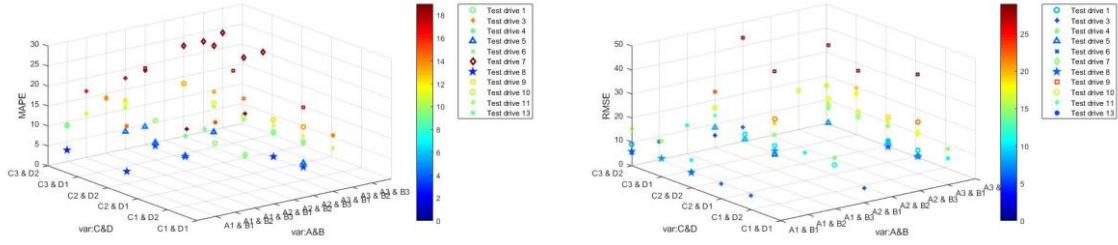


Figure9: MAPE (left) and RMSE (right) results for the 11 test drive with discharged battery and a quantile of 0.1

If we compare the occurrence of the optima in Fig. 10, both evaluation criteria show the same tendencies. Especially algorithms with parameter B1 are considered optimal. For example, parameter B3 only has values within the considered quantile of 0.1 in combination with parameter C3 and D2. Thus, our method allows us to quickly draw conclusions about the influence of the parameters on the accuracy of the calculation. If MAPE is used as the evaluation criterion, the other parameters also have a clear tendency since the combination with A3/C1/D2 has the highest occurrence. There is no such definitive optimum for RMSE since A2/C2/D2 has the same occurrence as A3/C1/D2 for the selected quantile.

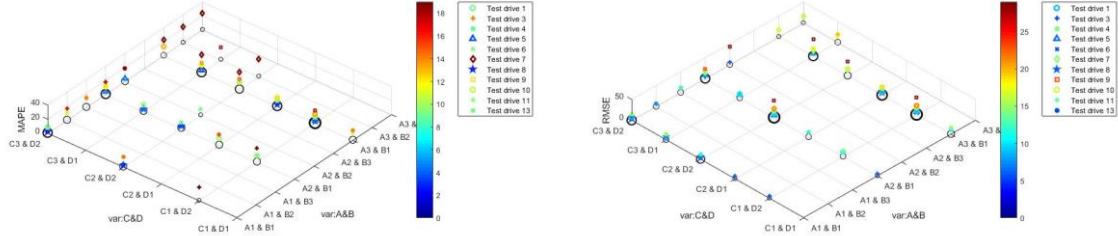


Figure10: MAPE (left) and RMSE (right) occurrences of optima of the 11 test drive with discharged battery and a quantile of 0.1

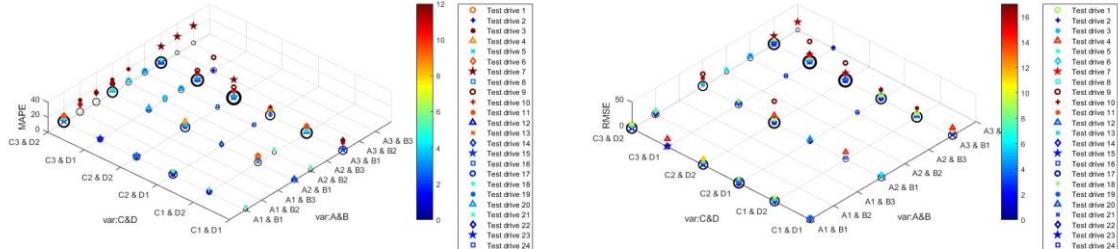


Figure11: MAPE (left) and RMSE (right) occurrences of optima of all test drives and a quantile of 0.06

If we consider all test drives, we reduced the number of values considered per test drive to 3, which means a quantile of 0.06. In this case B1 still has the highest occurrence, see Fig. 11. For the other parameters, however, the combinations A3/C2/D2 and A3/C3/D1 now dominate the previous combinations regarded as optimal. In addition, most of the shorter test drives result in a significantly lower MAPE or RMSE value than the ones with fully discharged battery. This suggests that equating the final values for an unknown drivable range due to remaining usable energy will falsify the results.

All the optimal combinations found still show dynamic changes, but these are significantly subdued compared to the original calculation, see for example Fig. 12 left. Further, for most test drives, the resulting range curves of the four possible optimal parameter combinations look quite similar. But the ones based on the test drives with a known driving distance A3/B1/C1/D2 or A2/B1/C2/D2 show a tendency to have lower start value errors. In case of an unknown range, differences in start values occur dependent on parameter A, see Fig. 12 right. However, in this case we can't say which one is correct.

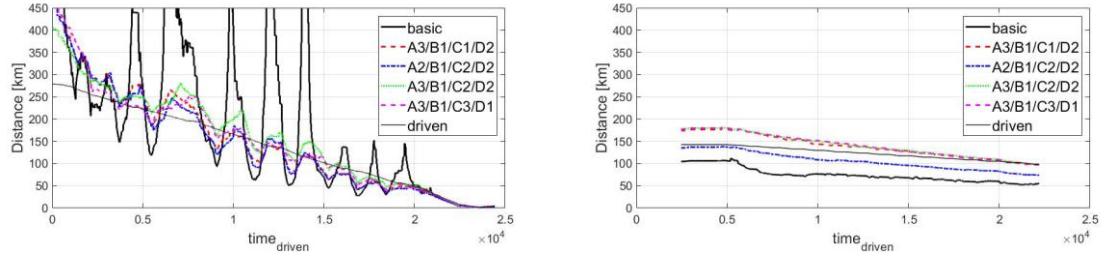


Figure12: Remaining range calculation of test drive 9 (left) and 25 (right) for the found optimal parameters

In summary, our methodology provides an easy way to compare different range calculations for a large number of test data. The results found by our approach are not the optimal calculation for a specific test set, but the optimal compromise for different boundary conditions and influencing variables. In addition, we can use our approach to make basic statements about the influence of various parameters on the range calculation. This helps in the targeted development of a reliable range calculation for the driver.

## 5 Conclusions and outlook

Accurate range prediction reduces range anxiety and could therefore have the same effect than an increase of the battery capacity. But the range calculation for an electric bus is difficult because there is not the “right” remaining range. In contrast to the diesel bus, the range does not only decrease, but can even significantly increase in the case of recuperation. However, for high reliability, the dynamics of the calculated range should basically follow the distance travelled as for a driver, it is easy to understand that the range is reduced by one kilometre when he has driven one. The challenge is to give the driver a reliable remaining range value while considering the dynamically changing consumption and on-board energy nonetheless.

In addition, the drivable distance is influenced by a variety of vehicle and environmental conditions, which may also affect the accuracy of the range calculation. Therefore, to assess the range calculation there should be a variety of test data used to ensure that the advantages or disadvantages found are not limited to specific environmental conditions. That is why we used 25 test drives with different driving behaviour and a variation of environmental conditions for our valuation. In 11 of these test drives, we discharged the battery fully, ten having a DOD of over 93%, and therefore know the drivable distance. In particular, changes in the route topology, driving dynamics and the use of auxiliaries seem to influence the accuracy of the range calculation.

As evaluation criteria we examined the AE, MAE, RMSE, RMSPE, MAPE and sMAPE between the range calculation and the inverse driven distance. When evaluating the range, criteria should be used that evaluate the entire calculation, not just start and end values, to be able to account for the dynamics while driving. Therefore, the AE was not considered further. The results for MAE and RMSE as well as the results for MAPE, sMAPE and RMSPE were similar or even identical in most cases, so we chose MAPE and RMSE as evaluation criteria for accuracy of the range calculation. We were able to show that both evaluation criteria are suitable for finding optimal range calculations. However, since the results found differ between the test drives and the two criteria, MAPE and RMSE alone are not sufficient for evaluation. That is why we have extended this approach. Instead of considering only the minimum value, we considered the x-quantile of the results, in our case the 5 smallest values with the 0.1-quantile, for each test run and assumed the algorithms with the highest occurrence of values as optimal. As a result, we obtained two possible optimal parameter combinations for an algorithm with reduced dynamics. Both calculations follow mainly, as desired, the driven distance. Since considering trips with remaining battery energy in the evaluation results in a poorer algorithm, if the distance travelled is to be used as a reference variable, we recommend using trips with a fully discharged battery to assess the accuracy of the range calculation, if possible.

In future research, we want to further explore the possibilities of combining multiple evaluation criteria to obtain better results even when considering trips with remaining energy. What is more, we want to improve our approach by adding additional evaluation criteria such as a dynamic tolerance window in case of changes of the available energy due to recuperation. Furthermore, we want to apply the findings outlined here to further improve the reliability of our range algorithms.

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